Experiments on the pressure drop created by a sphere settling in a viscous liquid

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Experimental measurements were made of the difference in pressure at large distances on either side of a spherical particle settling slowly along the axis of a long circular tube filled with viscous liquid. At low particle Reynolds numbers and for small sphere-cylinder diameter ratios, it was found that the product of the pressure difference and the cross-sectional area of the tube is equal to *twice* the drag on the particle, in accord with theory.

1. Introduction

The 'pressure drop' caused by the motion of a particle settling at small Reynolds numbers through an otherwise quiescent, viscous fluid contained within an infinitely long cylinder has been theoretically investigated on the basis of the Stokes (Brenner & Happel 1958, Brenner 1959) and Oseen (Brenner 1962) equations. For a particle settling along the axis of a circular tube of radius R_0, the pressure drop, ΔP, was found to be related to the drag, D, on the particle by the equation

$$\Delta PA = 2D, \tag{1}$$

where $A = \pi R_0^2$ is the cross-sectional area of the cylinder. The quantity ΔP is the dynamic pressure difference between any two planes at great distances on either side of the settling sphere, the pressure being greatest on that side of the particle towards which it advances. Equation (1) applies only when a/R_0 is small (a = sphere radius). The validity of this relation has been demonstrated for both the Stokes and Oseen régimes.

The discrepancy between this expression and the relation $\Delta PA = D$, which would be obtained by applying conventional momentum arguments to the limiting case of an 'unbounded' fluid (i.e. $a/R_0 = 0$) derives from the existence of a finite shearing force on the vertical walls of the cylinder—even if they be infinitely distant from the particle (Brenner 1962). This shearing force arises from a reverse flow near the walls, compensating for the fluid dragged along by the particle. The existence of this limiting wall force points up an important distinction between a truly unbounded fluid and one which is bounded at infinity.

The present work deals with the experimental confirmation of equation (1) for rigid spherical particles at low Reynolds numbers, $Re = 2aU\rho/\mu$ (U = terminal settling velocity, ρ = fluid density, μ = viscosity). A more complete description of the experiments and their interpretation is available elsewhere (Pliskin 1962).

2. Equipment and experiments

A sketch of the apparatus is shown in figure 1. In essence it consisted of three main parts: (i) a vertical, circular, glass column 10 ft. long and 6·03 in. inner diameter within which the spheres were dropped; (ii) a vertical, brass pipe 10 ft. long and 2 in. inner diameter joined at its base to the glass column (this

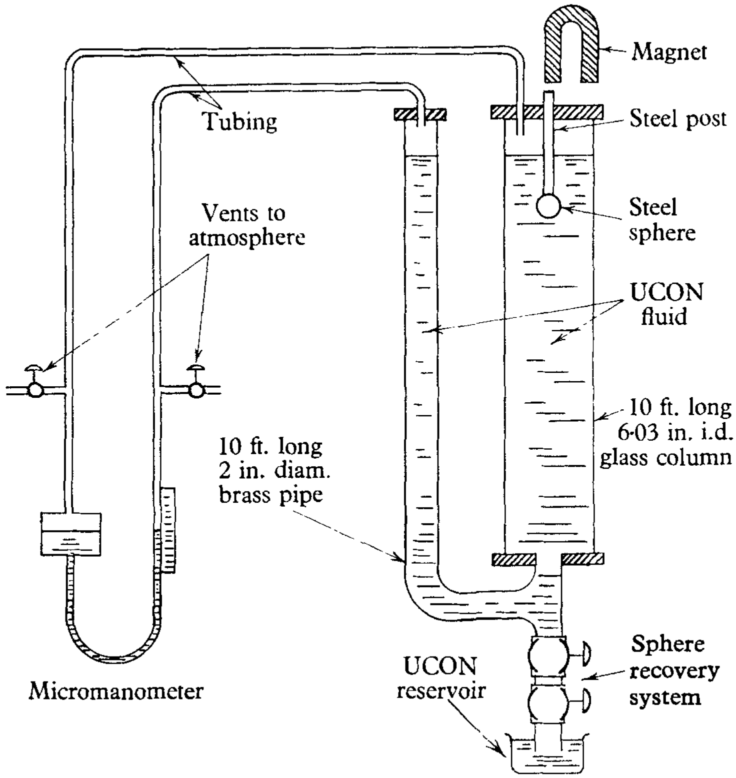

FIGURE 1. Sketch of apparatus.

column was employed to balance the hydrostatic pressure in the main column); (iii) a micromanometer with its low-pressure side attached by copper tubing to the top of the glass column and its high-pressure side attached to the top of the brass column.

The entire apparatus was housed entirely within a box-like, Plexiglass structure, the interior of which we attempted to maintain at constant temperature by continuously circulating air from a constant environment chamber.

During the experiments themselves, these three pieces of equipment formed a closed circuit, sealed-off from the atmosphere. This insured that fluctuations in local atmospheric pressure during the course of an experiment would not mask the small pressure differences being measured. Air-tightness of the system was tested by applying a vacuum to it and observing the rate of rise of pressure after turning off the vacuum pump.

The larger and smaller diameter columns were filled to within a few inches of the top with UCON 50-HB-5100 fluid (Union Carbide), a highly viscous commercial lubricant with a viscosity varying from 2550 to 2110 cP in the temperative range 23–27 °C and a density varying from 1·056 to 1·048 g/cm^3 in this range. By means of two gate valves, provision was made at the base of the main column for recovering the spheres without permitting air bubbles to enter the UCON fluid. Apart from the manometer itself, the remainder of the system contained air at essentially atmospheric pressure which served to transmit the pressures to the manometer.

A commercially available micromanometer was employed in the experiments (Model MM 3, Flow Corporation, Arlington, Massachusetts). In principle this instrument is a simple U-tube manometer, theoretically capable of resolving and measuring pressure differences as small as 10^{-4} in. of manometer fluid (butyl alcohol, density = 0·828 g/cm^3 at 25 °C). It achieves this unusual accuracy primarily by means of a mechanical null arranged so that the meniscus between the manometer fluid and air in one leg of the U-tube is in the same position for the 'zero' reading as it is with an imposed pressure difference. Thus, the complicating effects of surface tension cancel. Straightforward manometric principles were used to relate the micromanometer reading to the dynamic pressure difference across the column during the fall of a sphere.

To insure that the UCON fluid was Newtonian its rheological properties were investigated in a MacMichael viscometer at various shear rates in a range comparable to those encountered in the main experiments. Newtonian behaviour was observed. As a check on these Couette-type measurements, viscosities were also determined by observing the terminal settling velocities of various spheres in the glass column and treating the latter as a falling-ball viscometer. With appropriate wall corrections (Haberman & Sayre 1958) these viscosities agreed well with those obtained from the MacMichael apparatus.

In the pressure-drop experiments, spheres were released from rest *beneath* the surface of the UCON fluid in the glass column by a magnetic releasing device controlled from *outside* the system. In the case of steel spheres, the operation of the dropping mechanism is obvious from the sketch in figure 1. A modification of this was used for non-magnetic spheres. This technique assured that no extraneous disturbances were introduced into the experiment. Release of a sphere caused the liquid levels to fall in the main column and rise in the smaller column until they reached their equilibrium values appropriate to the pressure difference. Because of the high viscosity and large inertia of the fluid masses involved, the time response of the system to a suddenly applied pressure difference was approximately calculated. It was estimated that a 99 % approach to equilibrium would be attained within 20 sec. Qualitative observations of the response to the rapid pressure changes occurring at the beginning and end of an experiment indicated an even more favourable response time. Nevertheless, no runs were performed for which the time of passage of a sphere through the system would be less than about 20 sec.

An unexpected difficulty was encountered during the experiments. A continuous drift was observed in the 'zero' reading of the manometer, that is, with the entire system presumably at rest. (With the system vented to the atmosphere no drift was observed.) The drift never ceased but continued unabated for as long a period of time as we chose to observe it. Moreover, the drift was not unidirectional but, rather, underwent reversals over extended periods of time. It seems likely that the drift was due to non-uniformities in density of the UCON fluid arising from inadequate temperature control. Temperatures were controllable only to within 2–3 °C. Fortunately, the total change in the 'zero' reading during the length of time required for a run was not a major fraction of the pressure-drop reading itself, so the experiments were carried out anyway. To minimize the uncertainty in the pressure-drop reading caused by the drift

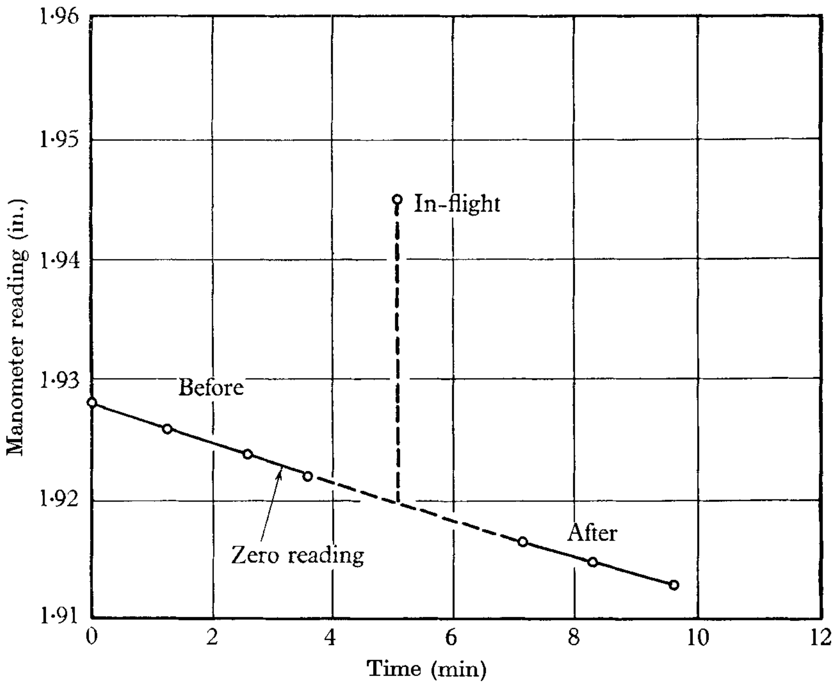

FIGURE 2. Drift in zero-point manometer reading.

the following procedure was adopted. Several 'zero' readings of the manometer were obtained at various times before and after each run. The exact time of the pressure-drop reading taken during the sphere's descent was also recorded. (This was normally done when the sphere was about midway down the column.) A plot of the series of 'before' and 'after' readings versus time furnished the 'zero' curve which could be smoothly interpolated. By these means the 'zero' reading at the exact time of the 'in-flight' reading was determined. This reading, when subtracted from the 'in-flight' reading, yielded the manometer reading appropriate to the pressure drop sought. The results for a typical run are shown in figure 2. In almost all instances the 'before' and 'after' readings lay on a continuous curve.

For the runs involving the denser spheres (spheres C, E, G and J in table 1), sufficient time was not available to measure the 'in-flight' reading by the previous techniques. Here, the procedure was modified. The system was vented to the atmosphere (so no drift occurred in the manometer) and the sphere released. When the sphere was about midway down the column the system was rapidly sealed-off from the atmosphere by turning a valve, and the time recorded. Thus the constant manometer reading, before sealing-off the system, provided the 'in-flight' reading. After the sphere came to rest at the bottom of the column, readings were taken of the manometer at various times. These 'zero' readings were then back-extrapolated to the time at which the system was shut-off from the atmosphere. By these means the true pressure-drop reading was obtained. This procedure is equivalent to obtaining only the 'in-flight' and 'after' readings in figure 2.

Sphere	Description	Diameter (in.)	Weight (g)
A	Aluminium	0·500	3·011
B	Aluminium	0·375	1·268
C	Bronze	0·375	3·965
D	Steel	0·250	1·046
E	Aluminium	0·736	9·489
F	Steel	0·282	1·484
G	Steel	0·312	2·007
H	Vinyl-steel†	0·975	10·954
I	Vinyl-steel†	1·230	21·725
J	Cellulose acetate-steel	1·510	46·305

† These were made by drilling a hole in the solid plastic spheres and inserting a steel rod in the interior.

TABLE 1. Physical properties of spheres

3. Results and conclusions

Data were obtained for ten different spheres ranging in size from 0·25 to 1·51 in., corresponding to a/R_0 values from 0·0414 to 0·250. The physical properties of these spheres are tabulated in table 1. The drag, D, on these spheres, required in subsequent calculations, is simply equal to the sphere weight corrected for the buoyancy of the UCON fluid. Five replicate measurements were made of the pressure drop, ΔP, for each sphere, except for spheres A and C where ten and six experiments, respectively, were performed. Several measurements were also made of the terminal settling velocity, U, for each sphere. Precise knowledge of these velocities was not critical to the investigation.

Values of the Reynolds number, $Re = 2aU\rho/\mu$, and the dimensionless ratio $\Delta PA/D$ ($A = 28{\cdot}6$ in.^{2}), calculated from the measured velocities and pressure drops, respectively, are tabulated in table 2. (The actual pressure drops ranged from about 0·004 to 0·07 in. of butyl alcohol in the experiments.) The Reynolds number tabulated for each sphere is only an average value as variations in temperature affect the Reynolds number. In general these Reynolds numbers are probably reliable to about 5 %. The rather good reproducibility of the pressure-drop measurements in replicate experiments, despite the drift referred to earlier, is evidenced by the relatively small standard deviations of the $\Delta PA/D$ ratios from the average for each sphere.

Sphere	a/R_0	Re	$\Delta PA/D$ (standard deviation)
A	0·0829	0·31	2·02 (0·09)
B	0·0622	0·12	2·24 (0·11)
C	0·0622	0·52	1·88 (0·04)
D	0·0415	0·15	2·18 (0·13)
E	0·122	0·71	1·94 (0·06)
F	0·0468	0·19	2·05 (0·18)
G	0·0517	0·26	2·07 (0·07)
H	0·162	0·34	2·04 (0·06)
I	0·204	0·57	2·15 (0·13)
J	0·250	1·6	1·89 (0·03)

TABLE 2. Experimental results.

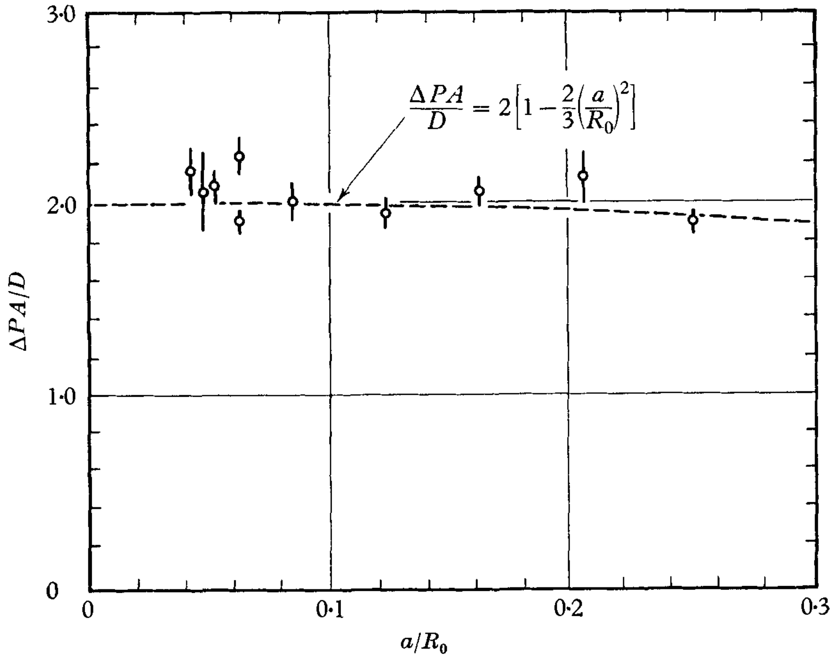

FIGURE 3. Pressure drop force-drag ratio *vs* sphere-to-cylinder diameter ratio.

Due to a slight wall effect, the $\Delta PA/D$ ratios in table 2 would not be expected to be exactly 2·0. According to theory† the correct relation in *creeping flow* for a sphere at the axis of a circular cylinder is

$$\Delta PA/D = 2[1-\tfrac{2}{3}(a/R_0)^2]+O(a/R_0)^3. \tag{2}$$

In the least favourable case, $a/R_0 = 0{\cdot}250$, the correction factor is only 4%. To compare theory and experiment, this wall-correction factor was applied to each of the 56 individual pressure-drop experiments. This yielded an average

† This wall correction factor follows from Brenner (1962) by setting $b = 0$ in equations (31)–(35). The wall-correction factor for non-circular cylinders can be obtained by similar arguments.

of $\Delta PA/D[1-\frac{2}{3}(a/R_0)^2] = 2{\cdot}06$ with a standard deviation of 0·15 from this average. This differs by only 3 % from the theoretical value of 2·0. As this ratio would be 1·0 if no shear forces existed at the wall, the data appear to confirm the theory. The pressure-drop data are also presented in figure 3 as a function of a/R_0. The dashed line corresponds to the theoretical values, equation (2).

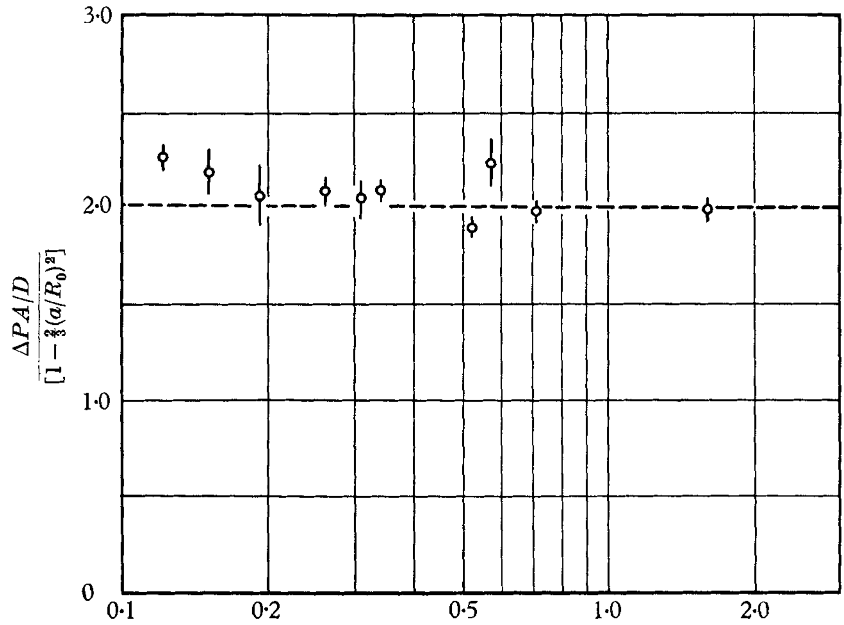

FIGURE 4. Corrected pressure drop force-drag ratio *vs* particle Reynolds number.

Figure 4 is a plot of the corrected pressure-drop ratio, $\Delta PA/D[1-\frac{2}{3}(a/R_0)^2]$, *vs* the particle Reynolds number. There appears to be no discernible effect of Reynolds number. This observation accords with theory. However, it must be pointed out that in the Reynolds-number range investigated, inertial effects would not normally be expected to be large anyway; hence, the data do not settle the issue of whether or not the $\Delta PA/D$ ratio is independent of Reynolds number, even at large Reynolds numbers, as originally speculated (Brenner 1962).

By eliminating the temperature-drift problem and employing a less viscous fluid such as water, there appears to be no fundamental reason why the Reynolds number range encompassed by the experiments cannot be extended to very much greater values than those reported upon here.

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